**CUBE HARMONIUS LABELING**

*Available online at* [*http://mesmampad.org/*](http://mesmampad.org/)*researchjournals.php*

And

1.Department of Mathematics, MES Mampad College, Mampad

2. Department of Mathematics, University of Calicut, Calicut

**ABSTRACT**: In this paper we have introduced a new harmonious labelling called cube harmoniouslabelling. A graph G(V,E) with n vertices and m edges is said to be a cube harmonious graph ifthere exist an injection f: V(G)*->*{1,2,3,............+1} such that the induced mapping :E(G){1,8,27,...........} defined by (uv)=(f(u)+f(v)) mod (+1) is a bijection.The resulting edgelabels and vertex labels are distinct. The function f is called a cube harmonious labelling of G. Here weprove that the path graph, stargraph, bistar graph and the comb graph are cube harmoniousGraphs.

KEYWORDS: Harmonious labelling, Bistar, Comb graph.

**INTRODUCTION**: In this paper, we consider finite, undirected, simple graph G (V, E) with n verticesand m edges. For notations and terminology we follow Bondy and Murthy [1]. Harmonious graphnaturally arose in the study by Graham and Sloane [3] off modular versions of additive base problems.Square graceful graphs were introduced in [4]. For a detailed survey on graph labelling we referto Gallian[2].We interested in the study of square harmonious labelling graphs by P.B.Sarasija andN.Adalin Beatress

***Definition***:

*A graph G(V,E) with n vertices and m edges is said to be a cube harmonious graph if*

*there exist an injection f: V(G) {1,2,3,............+1} such that the induced mapping : E(G) {1,8,27,...........} defined by =( f (u) +f (v) ) mod is a bijection.The resulting edgelabells and vertex labels are distinct. The function f is called a cube harmonious labelling of G.*

In this paper, we prove that the path graph, star graph, bistar graph and the graph

are cube harmonious graphs.

**Main Results**

***Theorem****: The star graph is acube harmonious graph for all n ≥* 2*:*

***Proof****: Let be a star graph with (n+1) vertices and m=n edges.*

*Let V()={,.......}.*

*Let be the centre vertex.*

*Let E () = { ,}.*

*Define an injective function f:V{1, 2, 3,............,+1} by*

*f()=+1 and*

*f()=i=1,2,........,n*

*(uv)=(f(u)+f(v)) mod (+1).*

*f induces a bijection.*

*Hence the star graph is a cube harmonious graph.*

**Theorem***: The bistar graph is a cube harmonious labelling graph.*

*Proof: Let be a bistar graph with n=p+q+2 vertices and m=p+q+1 edges.*

*Let V() = { }*

*Let B() = { ,uv}.*

*Define an injection function f:V) →{1,2,.......} by*

*f(u)=+1*

*f(v)=*

*f()=*

*f()=(+1,q.*

*= (f(u)+f(v)) mod (+1).*

*Hence the bistar graph is a cube harmonious graph.*

**Theorem***: Every path (n≥* 3*) is a cube harmonious graph.*

*Proof: Let be a path with n vertices and m=(n-1) edges. LetV() = { ,.......} andE()={}.*

*Define an injection f: V({1,2,3,............} by*

*f()=7, f()=1, f()=+1, f()=, f()=-3+3m*

*f()=f() +23+6(n-6),n≥6*

*and f()=23+6*x6+6 x 8+………+6 x (2*i –* 2)+6(*i –* 2)(*n -* 2*i*)+ *f() , 2i ≥* 8

*f()=f() -66 for n=7*

*f()=f() - (66*+6 x 5+6 x 6+……+6 x (*n -* 3)*, n ≥* 8

*f()=f() - (66+6*x7+6 x 9+ …….. +6( 2i - 1)+6(i+2)+6(*i*+3)*+.......+6(n-i)),*

*2i+1 ≥* 9*.*

*To prove f is a bijection in general we consider the following cases*

*()*=

*( )*

*( )* =

for n=7

*( )* =)

for n≥ 8

*( ) ≡* )

for n=8 and n≥8

*( ) ≡* )

*( ) ≡* ) for 2i *≥ 8, 2i+1≥9*

*Hence f induces a bijection*

*.*

*The edge labels are distinct.*

*Hence every path is a cube harmonious graph.*

**Theorem***: The comb graph s a cube harmonius graph.*

***Proof****: Let{ …….. } be the vertices of the path and{ ,....... be the pendent vertices at……… respectively.*

*Here m=2n - 1.*

*N is the total number of vertices of*

*Define an injection f : V () → {1,2,3,....... by*

*f()=7,f()=1, f()=*

*f(()=f()=*

*f()=f() +23+6(N – 6), n6*

*and f()=23+6*x6+6 x 8+……..+6 x(2i – 2)+6(i - 2)(*N –* 2i)+ *f() , 2i* 8

*f()=f() -66 for N=7*

*f()=f() - (66*+6 x 5+6 x 6+……….+6 x (*N -* 3)*), n*8

*f() =f() - (66+6*x7+6 x 9+ …+6(2*i -* 1)+6(*i*+2)+6(*i*+3)*+......+6(N-i)),*

*for 2i+1*9*.*

*f()=*

*f()=*

*f()=*

*f()=*

*f()= for i*

*The induced function :E() → {1, 8, 27,...........} is bijective.*

*Hence the comb graph is cube harmonious graph.*

REFERENCES

[1]. J.A.Bondy and U.S.R.Murthy, Graph theory with Applications, Macmillian, London,1976.

[2]. J.A.Gallian, A dynamic survey of graph labelling, the electronics J. of Combinatorics, 16, 2009.

[3]. R.L.Graham and N.J.A.sloane, on additive bases and harmonious graphs, SIAM J.Alg.Discretemath., 1(1980)382-404.

[4]. P.B.Sarasija and R.Binthiya, Even harmonious graphs with applications, International JournalOf Computer Science and Information security. Vol.9, No.7, (2011)161-163.

[5]. T.Tharmaraj and P.B.Sarasija, Square graceful graphs, international journal of MathematicsAnd Soft Computing Vol.4 No.1.[2014], 129-137

[6]. T.Tharmaraj and P.B.Sarasija, Some Square graceful graphs, international journal of MathematicsAnd Soft Computing Vol.5 No.1.[2014], 119-127

[7]. P.B.Sarasija and N.Adalin Beatress, Square harmonious graphs, International Journal of

Advanced Research in Science, Engineering and Technology Vol.3, issue 2, February (2016).

[8]. P.B.Sarasija and N.Adalin Beatress, Even - Odd harmonious graphs, international journal ofMathematics and Soft Computing Vol.5 No.1.[2015], 23-29.